Due: Oct 1

1. For functions $f, g : \mathbb{R} \to \mathbb{R}^3$, verify that

$$\frac{d}{dt}(f \times g) = \frac{df}{dt} \times g + f \times \frac{dg}{dt}.$$ 

2. Let $D \subset \mathbb{R}^n$ be open and for a function $f : D \to \mathbb{R}$, suppose there is a constant $M$ so that, for $i = 1, 2, \ldots, n$, and all points $p \in D$, we have

$$\left\| \frac{\partial f}{\partial x_i}(p) \right\| \leq M.$$

Prove that $f$ is continuous at every point of $D$.

HINT: Write the change in $f$ using partial derivatives, along the lines of the proof of the Differentiability Theorem.

It is not true that $f$ is differentiable at every point of $D$; the function in Question 3 of the last assignment is a counterexample.

3. Do Exercise S-7.73, that is, If $\mathbf{r} = a \cos \omega t + b \sin \omega t$, where $a$ and $b$ are any constant non-collinear vectors and $\omega$ is a constant scalar, prove that

$$(a) \mathbf{r} \times \frac{d\mathbf{r}}{dt} = \omega (a \times b), \quad (b) \frac{d^2\mathbf{r}}{dt^2} + \omega^2 \mathbf{r} = 0.$$ 

4. Do Exercise S-7.96 (b), that is, Prove that the radius of curvature at any point of the plane curve $y = f(x), z = 0$, where $f(x)$ is [twice] differentiable, is given by

$$\rho = \left| \frac{(1 + (y')^2)^{3/2}}{y''} \right|.$$