1. Suppose \( F, G : \mathbb{R}^n \rightarrow \mathbb{R} \) satisfy \( \lim_{x \to a} F(x) = L \) and \( \lim_{x \to a} G(x) = M \). Prove that
\[
\lim_{x \to a} F(x)G(x) = LM.
\]

**Hint:** Look at the proof given in class of the analogous result for sums.

2. Let \( S^* \subset \mathbb{R}^n \) be the set of all limit points of \( S \subset \mathbb{R}^n \). Show that \( S \cup S^* \) is closed, i.e., contains all of its limit points.

3. Consider the function \( F : \mathbb{R}^3 \rightarrow \mathbb{R} \) given by \( F(x, y, z) = x^2 + y^2 + z^2 \).
   (a) Find the differential of \( F \) at \( a = (3, 2, 6) \), \( dF_a \), which is a linear transformation from \( \mathbb{R}^3 \) to \( \mathbb{R}^3 \).
   (b) Using the differential, find an approximate value for \( 3.02^2 + 1.97^2 + 5.98^2 \).

4. If \( F : \mathbb{R}^n \rightarrow \mathbb{R}^m \) is linear, show that, for each point \( a \in \mathbb{R}^n \), \( F \) is differentiable at \( a \) and the differential \( dF_a \) equals \( F \).

5. Suppose that \( F : \mathbb{R}^n \rightarrow \mathbb{R}^m \) and \( G : \mathbb{R}^n \rightarrow \mathbb{R}^k \) are both differentiable at \( a \in \mathbb{R}^n \). If \( H : \mathbb{R}^n \rightarrow \mathbb{R}^{m+k} \) is given by \( H(x) = (F(x), G(x)) \) then show directly from the definition that \( H \) is differentiable at \( a \).