Math 496-004 **Assignment # 2** **3 Sept 08**

Due: Sept 10th

1. Do Exercise S-6.53 (b), that is, prove that $\lim_{(x,y)\to(2,1)}(xy-3x+4) = 0$ using the definition.

2. Do Exercise S-6.58, that is, does $\lim_{(x,y,z)\to(0,0,0)} \frac{4x+y-3x}{2x-5y+2z}$ exist? Justify your answer.

3. Consider the function $F : \mathbb{R}^2 \to \mathbb{R}$ given by

$$F(x,y) = \begin{cases} \dfrac{2x^2y}{x^4+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

   (a) Show, for any straight line L through $(0,0)$, the limit of F along the line L is 0.

   (b) Show that, for the function $\phi : \mathbb{R} \to \mathbb{R}^2 : t \mapsto (t, t^2)$, $\lim_{t\to 0} F(\phi(t)) = 1$.

   (c) Is it true that $\lim_{(x,y)\to(0,0)} F(x,y) = 0$? Justify your answer.

4. Suppose $F, G : \mathbb{R}^n \to \mathbb{R}$ satisfy $\lim_{x\to a} F(x) = L$ and $\lim_{x\to a} G(x) = M$. Prove that

$$\lim_{x\to a} F(x)G(x) = LM.$$

   Hint: Look at the proof given in class of the analogous result for sums.

5. Do Exercise S-4.52 (b) & (c), that is, using differentials, compute approximate values for each of $\ln(1.12)$ and $\sqrt[5]{36}$.