Due: Sept 10th

1. Do Exercise S-6.53 (b), that is, prove that \( \lim_{(x,y) \to (2,1)} (xy - 3x + 4) = 0 \) using the definition.

2. Do Exercise S-6.58, that is, does \( \lim_{(x,y,z) \to (0,0,0)} \frac{4x + y - 3x}{2x - 5y + 2z} \) exist? Justify your answer.

3. Consider the function \( F: \mathbb{R}^2 \to \mathbb{R} \) given by

\[
F(x, y) = \begin{cases} 
\frac{2x^2 y}{x^4 + y^2} & \text{if } (x, y) \neq (0, 0), \\
0 & \text{if } (x, y) = (0, 0).
\end{cases}
\]

(a) Show, for any straight line \( L \) through \((0,0)\), the limit of \( F \) along the line \( L \) is 0.
(b) Show that, for the function \( \phi: \mathbb{R} \to \mathbb{R}^2: t \mapsto (t, t^2) \), \( \lim_{t \to 0} F(\phi(t)) = 1 \).
(c) Is it true that \( \lim_{(x,y) \to (0,0)} F(x, y) = 0 \)? Justify your answer.

4. Suppose \( F, G: \mathbb{R}^n \to \mathbb{R} \) satisfy \( \lim_{x \to a} F(x) = L \) and \( \lim_{x \to a} G(x) = M \). Prove that

\[
\lim_{x \to a} F(x)G(x) = LM.
\]

HINT: Look at the proof given in class of the analogous result for sums.

5. Do Exercise S-4.52 (b) & (c), that is, using differentials, compute approximate values for each of \( \ln(1.12) \) and \( \sqrt[3]{36} \).