1. Consider the IVP $\dfrac{dy}{dt} = y^2 - t - 1$, $y(1) = 2$. Use Euler's method with $\Delta t = .1$ to find the points (t_1, y_1) and (t_2, y_2) and write down the formula for y_{n+1} in general.

   Solution. The points are $(t_1, y_1) = (1.1, 2.2)$ and $(t_2, y_2) = (1.2, 2.474)$. In general, $y_{n+1} = y_n + .1(y_n^2 - t_n - 1)$.

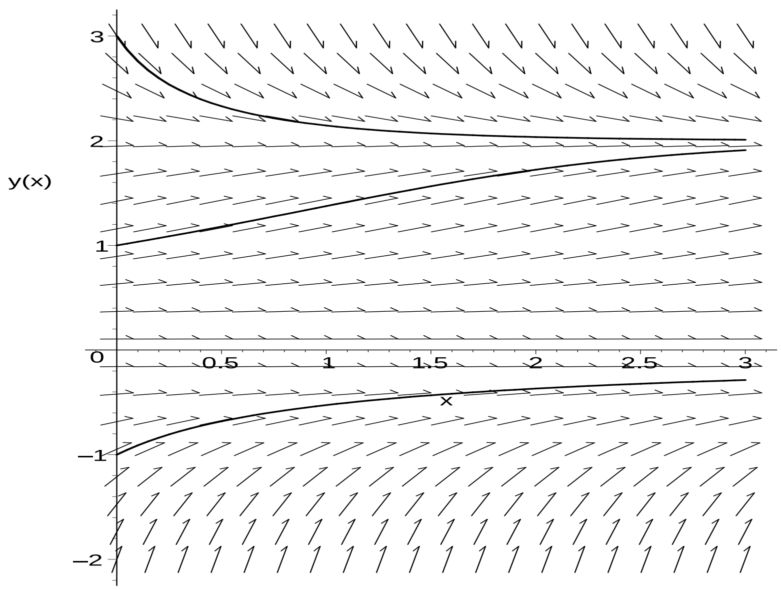
2. Consider the DE $y' = Ky^2 - y^3$ where $K > 1$.

   (a) Sketch the phase phase line and identify the equilibrium solutions.

   (b) Sketch the solutions to the DE satisfying $y(0) = -1$, $y(0) = 1$ and $y(0) = K + 1$.

   (c) Describe the possible values of $\lim_{t\to\infty} y(t)$.

   Solution. The equilibrium solutions are $y = 0$ and $y = K$. A direction field plane, with the solutions marked, is



   For (c), the possible values of $\lim_{t\to\infty} y(t)$ are 0 and K.

3. A parachutist whose mass is 50 kg drops from a helicopter and falls toward the ground under the influence of gravity. Assume the force due to air resistence is proportional to the velocity of the parachutist, with proportionality constant $b = 20N - s/m$ when the chute is closed. The chute does not open until the velocity of the parachutist reaches 20 m/s.

   (a) Does the chute open, and if so, when? (You may assume that the helicopter is at a high enough altitude.)

   (b) If the mass of the parachutist is 40kg, does the chute open and if so, when?

Solution. Let v be the velocity of the parachutist, then the IVP is

$$m\frac{dv}{dt} = mg - 20v, \qquad v(0) = 0$$

For part (a), we have $m = 50$ kg. Solving the DE gives $v(t) = 49/2(1 - e^{-2t/5})$ and $v(t) = 20$ when $t = 4.236$ seconds.

For part (b), we have $m = 40$ kg. Without solving the DE, we can work out the terminal velocity, which is 19.8 m/s. Since this is less than 20, the chute will never open. Alternatively, you can solve the DE to get $v(t) = 98/5(1 - e^{-t/5})$ and observe that the values of this function are at most $98/5 = 19.8$ m/s.

4. Consider the DE: $t^2y'' + (t-1)y' + (t-2)y = 0$. What is the largest interval around the initial conditions $y(1) = 1$, $y'(1) = 3$ on which the solution is certain to exist?

   Solution. $(0, +\infty)$.

5. Find the interval on which the IVP $x(x-4)y'' + 3xy' + 2y = x^2$, $y(1) = 2$, $y'(1) = 3$ has a unique solution. Do not solve it!

   Solution. $(0, 4)$.

6. Show that the two functions are linearly independent solutions of the DE and find the linear combination that satisfies the given initial conditions: $y_1(t) = e^t$, $y_2(t) = t^2 + 2t + 2$, $ty'' - (t+2)y' + 2y = 0$, $y(1) = 4$, $y'(1) = 6$.

   Solution. $y = (14/e)e^t - 2(t^2 + 2t + 2)$.

7. Find the general solutions to the following DEs:

   (a) $y'' - 7y' + 12y = 0$,

   (b) $x^2y'' + 3xy' = 2$,

   (c) $y^3y'' = 1$,

   Solution. For (a), $y = C_1e^{3x} + C_2e^{4x}$; for (b), $y = \ln x + Ax^{-2} + B$; for (c), $Ay^2(B - x) = 1$.