**Math 221-601** **Review for Test #1** **18 July 07**

This was a two hour test; we have not yet done Euler's method.

. 13 1. For each of the following IVPs, does an appropriate Existence and Uniqueness Theorem apply? Why or why not? If yes, what does the theorem say about the solution of the IVP.

   (a) $ty' + y = 2, \quad y(1) = 0$
   (b) $y' = (y^2 + t^2)^{1/3}, \quad y(0) = 0$

. 12 2. Solve $(e^x \sin y - 3x^2)dx + (e^x \cos y + y^{-2/3}/3)dy = 0$.

. 15 3. Consider the IVP $y' = y^2$, $y(0) = 1$.

   (a) Verify (showing your work) that $y(t) = 1/(1-t)$ is a solution of the IVP.
   (b) Using Euler's method with step size 1/2, find approximations to the solution at $t = 1/2$ and $t = 1$.
   (c) What is the error between the exact and approximate solutions at $t = 1/2$ and $t = 1$?

. 15 4. Solve $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{y^2 + x^2y^2}}, \qquad y(0) = 2.$

. 20 5. Consider a tank containing 50 L of water with 10 kg of salt dissolved in the water. Salt water containing $K$ kg/L of salt (where $K$ is a constant) are pumped into the tank at a rate of 5 L/min and well-mixed water is drained from the tank at the same rate.

   (a) Set up an IVP for this situation.
   (b) If there is to be 7 kg of salt in the tank after 10 minutes, what should the value of $K$ be? (Hint: since $K$ is a constant, you can solve the IVP using standard methods.)
   (c) For your value of $K$, what is the eventual amount of salt in the tank? That is, what is the limit as $t \to \infty$?

. 25 6. We model a population of mice on an island using an exponential model. After 10 years on the island, the population doubles to 50,000 mice.

   (a) Set up an IVP for the population of mice and find a function $M(t)$ for the population of mice since they were introduced to the island.
   (b) At this point (i.e., 10 years after the mice arrive), the islanders introduce some cats, who eat 6,000 mice each year. Set up a new IVP for the population of mice after the cats arrive.
   (c) Solve the IVP from the previous part and compare the population 10 years after the cats arrive to what it would have been without the cats.
   (d) Sketch a direction field for your DE (hint: the right hand side of the DE doesn't depend on $t$) and predict the behavior of the system.
   (e) (bonus question) Is the DE a reasonable population model? Why or why not?