Math 221-601 **Answers to Quiz # 5** **14 Aug '07**

. 10 1. Using Laplace Transforms, solve $x'' + 2x' + 2x = e^{-2t}$, $x(0) = 1$, $x'(0) = 1$.

Solution. Applying the Laplace Transform, we have

$$\begin{aligned} s^2X - s - 1 + 2(sX - 1) + 2X &= \frac{1}{s+2} \\ (s^2 + 2s + 2)X &= \frac{1}{s+2} + s + 3 \\ (s^2 + 2s + 2)X &= \frac{s^2 + 5s + 7}{s+2} \\ X &= \frac{s^2 + 5s + 7}{(s+2)(s^2 + 2s + 2)} \end{aligned}$$

Completing the square, $s^2 + 2s + 2 = (s+1)^2 + 1$ and so

$$\begin{aligned} \frac{s^2 + 5s + 7}{(s+2)(s^2 + 2s + 2)} &= \frac{A}{s+2} + \frac{B(s+1) + C}{(s+1)^2 + 1} \\ s^2 + 5s + 7 &= A(s^2 + 2s + 2) + B(s+1)(s+2) + C(s+2) \end{aligned}$$

Letting $s = -2$, we have $1 = 2A$ so $A = 1/2$. Letting $s = -1$, we have $3 = A + C$, so $C = 5/2$. Finally, letting $s = 0$, we have $7 = 2A + 2B + 2C$, so $B = (7 - 1 - 5)/2 = 1/2$. Thus,

$$X(s) = \frac{s^2 + 5s + 7}{(s+2)(s^2 + 2s + 2)} = \frac{1/2}{s+2} + \frac{1/2(s+1) + 5/2}{(s+1)^2 + 1}$$

Using the inverse Laplace transform,

$$x(t) = \frac{1}{2}e^{-2t} + \frac{1}{2}e^{-t}\cos t + \frac{5}{2}e^{-t}\sin t.$$