Math 221-601 **Answers to Quiz # 4** **8 Aug '07**

5 1. Find the equilibrium solutions (a.k.a. critical points) to the following system:

$$\frac{dx}{dt} = x(5x + 2y - 10), \qquad \frac{dy}{dt} = y(4 - x - y).$$

Solution. Solving the equation $x(5x + 2y - 10) = 0$ we have either $x = 0$ or $5x + 2y = 10$. Solving the equation $y(4 - x - y) = 0$ we have either $y = 0$ or $x + y = 4$.

Thus, there are four equilibrium points. Three of them come from setting x or y (or both) equal to zero: $(0, 0)$, $(0, 4)$, $(2, 0)$

The remaining one comes from solving the pair of equations $5x + 2y = 10$, $x + y = 4$. Taking the first minus twice the second gives $3x = 2$, so $x = 2/3$. Substituting this into $x + y = 4$ gives $y = 10/3$. Thus, the final equilibrium point is $(2/3, 10/3)$.

5 2. Find two linearly independent eigenvectors of the matrix $\begin{bmatrix} -3 & 2 \\ -3 & 4 \end{bmatrix}$, given that the eigenvalues are 3 and -2.

Solution. For $\lambda = 3$, we solve the system

$$\begin{bmatrix} -6 & 2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

This gives one equation, $-3v_1 + v_2 = 0$, we take $v_1 = 1$ and $v_2 = 3$. Thus, the eigenvector is $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

For $\lambda = -2$, we solve the system

$$\begin{bmatrix} -1 & 2 \\ -3 & 6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

This gives one equation, $-v_1 + 2v_2 = 0$, we take $v_2 = 1$ and $v_1 = 2$. Thus, the eigenvector is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.