

- 4 1. Solve $y^{(3)} - 3y'' = 0$, $y(0) = 10$, $y'(0) = 7$, $y''(0) = 18$.

Solution. The characteristic equation is $r^3 - 3r^2 = 0$, which has roots 0, 0, 3. Thus, the general solution of $y''' - 3y'' = 0$ is $y = C_1 e^{3x} + C_2 + C_3 x$. Using the initial conditions, we have

$$\begin{aligned} 10 &= y(0) = C_1 + C_2 \\ 7 &= y'(0) = 3C_1 + C_3 \\ 18 &= y''(0) = 9C_1 \end{aligned}$$

From the last equation, $C_1 = 2$. Substituting this into the second equation gives $7 = 6 + C_3$, so $C_3 = 1$. Substituting $C_1 = 2$ into the first equation gives $10 = 3 + C_2$, so $C_2 = 8$. Thus, the solution is

$$y = 2e^{3x} + 8 + x.$$

- 6 2. Find the general solution of $y'' + 9y = 3 \sin 2x$.

Solution. The general solution of $y'' + 9y = 0$ is $C_1 \cos 3x + C_2 \sin 3x$. The annihilator of $3 \sin 2x$ is $D^2 + 4$, and applying this to our DE gives

$$(D^2 + 4)(D^2 + 9)[y] = (D^2 + 4)[3 \sin 2x] = 0.$$

The solution of this DE is

$$C_1 \cos 3x + C_2 \sin 3x + C_3 \cos 2x + C_4 \sin 2x.$$

and the last two terms are not in the general solution of the original DE, so they constitute our guess.

Notice that, if $y = A \cos 2x + B \sin 2x$, then $y' = -2A \sin 2x + 2B \cos 2x$ and $y'' = -4A \cos 2x - 4B \sin 2x$. Putting these into the DE, we have

$$\begin{aligned} (-4A \sin 2x - 4B \cos 2x) + 9(A \cos 2x + B \sin 2x) &= 3 \sin 2x \\ 5A \sin 2x + 5B \cos 2x &= 3 \sin 2x \end{aligned}$$

Thus, $A = 3/5$ and $B = 0$, so the particular solution is $y = 3/5 \sin 2x$ and the general solution of $y'' + 9y = 3 \sin 2x$ is

$$y = \frac{3}{5} \sin 2x + C_1 \cos 3x + C_2 \sin 3x.$$