Math 221-601 Answers to Quiz # 1 19 July '07

. 4 1. Find an explicit general solution to $\dfrac{dy}{dx} = y \cos x$.

Solution. We separate variables and integrate to obtain

$$\begin{aligned} \frac{dy}{y} &= \cos x \, dx \\ \int \frac{dy}{y} &= \int \cos x \, dx \\ \ln |y| &= \sin x + C \\ |y| &= e^{\sin x + C} = K e^{\sin x} \end{aligned}$$

where $K = e^C$.

. 6 2. At noon, a car starts from rest at point A and proceeds with constant acceleration along a straight road towards a point B. If the car reaches B at 12:50 P.M. with a velocity of 60 mi/h, what is the distance from A to B?

Solution. Let k be the constant acceleration and ℓ the distance from A to B. Then we know the following facts about the position function $p(t)$, where p is measured in miles from A and t in hours.

$$p''(t) = k, \qquad p(5/6) = \ell, \qquad p'(5/6) = 60, \qquad p'(0) = 0.$$

We also know that $p(0) = 0$, since the car is starting from A. Integrating the DE, we have $p'(t) = kt + C$ and using $p'(0) = 0$, we see that $0 = p'(0) = k0 + C$, so $C = 0$. Integrating $p'(t) = kt$, we have $p(t) = kt^2/2 + C$ and using $p(0) = 0$, we have $0 = p(0) = k0^2/2 + C$, so $C = 0$ again. Using this formulae for p' and p, the remaining two equations become

$$60 = p'(5/6) = \frac{5k}{6}, \qquad \ell = p(5/6) = \frac{k(5/6)^2}{2} = 25k/72.$$

Solving the first equation gives $5k = 360$ so $k = 72$ mi/h^2 and substituting this into the second equation gives

$$\ell = \frac{25(72)}{72} = 25 \text{ mi}.$$