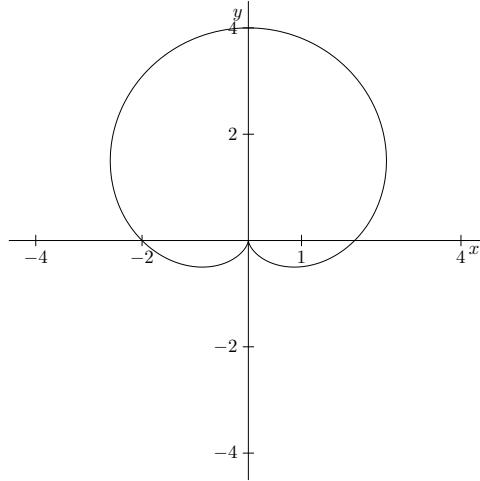


- 10 1. Graph the polar equation  $r = 2 + 2 \sin \theta$  on the (polar) axes below. List any symmetries that it displays, giving algebraic justifications for your answers.

*Solution.*



This is a cardioid with the main bulb on the upper half of the plane. To determine the types of symmetry, we first replace  $\theta$  with  $-\theta$  and obtain  $2 + 2 \sin \theta \rightarrow 2 + 2 \sin(-\theta) = 2 + 2 \sin \theta = r, \neq -r$ . Thus we have  $x$ -axis symmetry but not symmetry about the  $y$ -axis. Also, if we replace  $r$  with  $-r$ , we get  $r = 2 + 2 \sin \theta \rightarrow -r = -(2 + 2 \sin \theta) = -2 - 2 \sin \theta \neq 2 + 2 \sin \theta$ , which suggests that the graph does not have symmetry about the origin.