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1. Does  $\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n^2-4}}$  converge or diverge?

*Solution.* We apply the limit comparison test.

Let  $a_n = \frac{3}{n\sqrt{n^2-4}}$ , we expect  $a_n$  to behave like  $\frac{3}{n\sqrt{n^2}} = \frac{3}{n^2}$ , so we let

$$b_n = \frac{1}{n^2}.$$

Since  $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$  converges by  $p$ -series and

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{3n^2}{n\sqrt{n^2-4}} = \lim_{n \rightarrow \infty} \frac{3}{\sqrt{1-4/n^2}} = 3 > 0,$$

$\sum_{n=1}^{\infty} \frac{3}{n\sqrt{n^2-4}}$  converges by the limit comparison test.

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2. Does  $\sum_{n=1}^{\infty} \frac{(n+2)(n+3)}{(n+1)!}$  converge or diverge?

*Solution.* We apply the ratio test.

Let  $a_n = \frac{(n+2)(n+3)}{(n+1)!}$ , then  $a_{n+1} = \frac{(n+3)(n+4)}{(n+2)!}$ , and

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} &= \lim_{n \rightarrow \infty} \frac{(n+3)(n+4)(n+1)!}{(n+2)(n+3)(n+2)!} \\ &= \lim_{n \rightarrow \infty} \frac{n+4}{(n+2)(n+2)} \\ &= \lim_{n \rightarrow \infty} \frac{n+4}{n^2+4n+4} \\ &= \lim_{n \rightarrow \infty} \frac{1+4/n}{n+4+4/n} = 0 < 1, \end{aligned}$$

the series converges by the ratio test.