. 10 1. Evaluate $\displaystyle\int_{-1}^{2} \frac{1}{\sqrt{|x-1|}}dx$.

Hint: for $x > 1$, $|x-1| = x-1$ while for $x < 1$, $|x-1| = 1-x$.

Solution. The denominator goes to zero at $x = 1$. Rewrite the integral as

$$
\begin{aligned}
&= \int_{-1}^{1} \frac{1}{\sqrt{|x-1|}}dx + \int_{1}^{2} \frac{1}{\sqrt{|x-1|}}dx, \\
&= \lim_{a\to 1^-} \int_{-1}^{a} \frac{1}{\sqrt{|x-1|}}dx + \lim_{a\to 1^+} \int_{a}^{2} \frac{1}{\sqrt{|x-1|}}dx, \\
&= \lim_{a\to 1^-} \int_{-1}^{a} \frac{1}{\sqrt{1-x}}dx + \lim_{a\to 1^+} \int_{a}^{2} \frac{1}{\sqrt{x-1}}dx
\end{aligned}
$$

Use the substitution $u = 1-x, du = -dx$ for the first integral and $u = x-1, du = dx$ for the second integral to obtain:

$$
\begin{aligned}
&= \lim_{a\to 1^-} \int_{2}^{1-a} \frac{-du}{\sqrt{u}} + \lim_{a\to 1^+} \int_{a-1}^{1} \frac{du}{\sqrt{u}}, \\
&= \lim_{a\to 1^-} -2\sqrt{u}\Big|_{2}^{1-a} + \lim_{a\to 1^+} 2\sqrt{u}\Big|_{a-1}^{1}, \\
&= \lim_{a\to 1^-} -2[\sqrt{1-a} - \sqrt{2}] + \lim_{a\to 1^+} 2[1 - \sqrt{a-1}], \\
&= 2\sqrt{2} + 2
\end{aligned}
$$