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1. Evaluate the integral $\int \frac{v^2}{(1-v^2)^{1/2}} dv$.

Recall that $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ and that $\sin 2\theta = 2 \sin \theta \cos \theta$.

Solution. We set $v = \sin \theta$; then $dv = \cos \theta d\theta$ and $\cos \theta = \sqrt{1-v^2}$, according to the table of substitutions given in class.

Using this substitutions, we get

$$\begin{aligned} \int \frac{v^2}{(1-v^2)^{1/2}} dv &= \int \frac{\sin^2 \theta \cos \theta}{\cos \theta} d\theta \\ &= \int \sin^2 \theta d\theta \\ &= \int \frac{1 - \cos 2\theta}{2} d\theta \\ &= \frac{1}{2} \theta - \frac{\sin 2\theta}{4} + C \\ &= \frac{1}{2} \arcsin v - \frac{\sin \theta \cos \theta}{2} + C \\ &= \frac{1}{2} \arcsin v - \frac{v\sqrt{1-v^2}}{2} + C \end{aligned}$$