

Math 107-250/350 - Analytic Geometry & Calculus I 2nd Semester, '06-'07
Improper Integral Example

Evaluate $\int_{-1}^1 |x|^{-1/2} dx$.

Because $|x|^{-1/2}$ has a vertical asymptote at $x = 0$, we split the integral into two limits of proper integrals, namely

$$\int_{-1}^1 |x|^{-1/2} dx = \lim_{d \rightarrow 0^-} \int_{-1}^d |x|^{-1/2} dx + \lim_{g \rightarrow 0^+} \int_g^1 |x|^{-1/2} dx.$$

For the first term, we have

$$\begin{aligned} \lim_{d \rightarrow 0^-} \int_{-1}^d |x|^{-1/2} dx &= \lim_{d \rightarrow 0^-} \int_{-1}^d (-x)^{-1/2} dx \\ &= \lim_{d \rightarrow 0^-} - \left(2(-x)^{1/2} \Big|_{-1}^d \right) \\ &= - \left(\lim_{d \rightarrow 0^-} 2\sqrt{-d} - 2 \right) = 2. \end{aligned}$$

For the second term, we have

$$\begin{aligned} \lim_{g \rightarrow 0^+} \int_g^1 |x|^{-1/2} dx &= \lim_{g \rightarrow 0^+} \int_g^1 x^{-1/2} dx \\ &= \lim_{g \rightarrow 0^+} 2x^{1/2} \Big|_g^1 \\ &= - \lim_{g \rightarrow 0^+} 2 - 2\sqrt{g} = 2. \end{aligned}$$

Adding the terms together,

$$\int_{-1}^1 |x|^{-1/2} dx = 4.$$