Math 107-350/550 - Analytic Geometry & Calculus I **2nd Semester, '06-'07**

Challenge Integrals

Evaluate $\displaystyle\int \frac{1}{1+\sqrt{1+x}}\,dx$.

Use the substitution $u = \sqrt{x+1}$, so that $u^2 - 1 = x$ and $2u\,du = dx$. Then

$$\int \frac{1}{1+\sqrt{1+x}} = \int \frac{2u}{1+u}\,du$$

and now notice that $\dfrac{2u}{1+u} = \dfrac{2(1+u)}{1+u} - \dfrac{2}{1+u}$. You can also do polynomial long division to get this. Either way, we have

$$\begin{aligned} &= \int 2 - \frac{2}{1+u}\,du \\ &= 2u - 2\ln(1+u) + C \\ &= 2\sqrt{1+x} - 2\ln(1+\sqrt{1+x}) + C \end{aligned}$$

Evaluate $\displaystyle\int \frac{1}{\sqrt{x}+x^{1/4}}\,dx$.

Use the substitution $u = x^{1/4}$ so that $u^4 = x$ and $4u^3\,du = dx$. Then

$$\begin{aligned} \int \frac{1}{\sqrt{x}+x^{1/4}}\,dx &= \int \frac{4u^3}{u^2+u}\,du \\ &= \int \frac{4u^2}{u+1}\,du \end{aligned}$$

Using polynomial long division, $\frac{u^2}{1+u} = u - 1 + \frac{1}{u+1}$,

$$\begin{aligned} &= 4\int u - 1 + \frac{1}{u+1}\,du \\ &= 4\left(\frac{u^2}{2} - u + \ln(1+u)\right) + C \\ &= 2\sqrt{x} - 4x^{1/4} + 4\ln(1+x^{1/4}) + C \end{aligned}$$