

Math 107-250/350 - Analytic Geometry & Calculus I 2nd Semester, '06-'07
Arctan Examples

Evaluate $\int \frac{2}{x^2 + 6x + 15} dx$.

Notice that

$$x^2 + 6x + 15 = x^2 + 6x + 9 + 6 = (x + 3)^2 + 6$$

so the quadratic denominator is irreducible. Then we rewrite the fraction as

$$\frac{2}{x^2 + 6x + 15} = \frac{2/6}{\left(\frac{x+3}{\sqrt{6}}\right)^2 + 1}$$

Using this and the substitution $u = (x + 3)/\sqrt{6}$, we obtain

$$\begin{aligned} \int \frac{2}{x^2 + 6x + 15} dx &= \int \frac{1/3}{\left(\frac{x+3}{\sqrt{6}}\right)^2 + 1} dx \\ &= \int \frac{\sqrt{6}/3}{u^2 + 1} du \\ &= \frac{\sqrt{6}}{3} \arctan u + C \\ &= \frac{\sqrt{6}}{3} \arctan \left(\frac{x + 3}{\sqrt{6}} \right) + C \end{aligned}$$

Evaluate $\int \frac{5}{x^2 - 4x + 8} dx$.

Notice that

$$x^2 - 4x + 8 = x^2 - 4x + 4 + 4 = (x - 2)^2 + 4$$

so the quadratic denominator is irreducible. Then we rewrite the fraction as

$$\frac{5}{x^2 - 4x + 8} = \frac{5/4}{\left(\frac{x-2}{2}\right)^2 + 1}$$

Using this and the substitution $u = (x - 2)/2$, we obtain

$$\begin{aligned} \int \frac{5}{x^2 - 4x + 8} dx &= \int \frac{5/4}{\left(\frac{x-2}{2}\right)^2 + 1} dx \\ &= \int \frac{5/2}{u^2 + 1} du \\ &= \frac{5}{2} \arctan u + C \\ &= \frac{5}{2} \arctan \left(\frac{x - 2}{2} \right) + C \end{aligned}$$