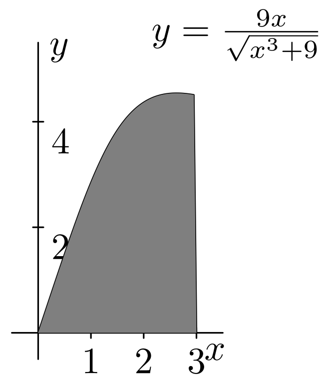
. 5 1. Set up an integral to find the volume of the solid generated by revolving the region bounded by $y = \dfrac{9x}{\sqrt{x^3+9}}$, the x-axis and the vertical line $x = 3$ about the y-axis. You do not need to evaluate the integral.

Solution.



The shell is parallel to the axis of revolution. So the shell height is height $= \dfrac{9x}{\sqrt{x^3+9}}$ and the shell radius (i.e. the distance from the axis of revolution) is radius $= x$. The shell thickness variable is x, so the limits of integrations are $a = 0$ and $b = 3$. The volume of the solid is given by:

$$\begin{aligned} V &= 2\pi \int_a^b \begin{pmatrix} \text{shell} \\ \text{radius} \end{pmatrix} \begin{pmatrix} \text{shell} \\ \text{height} \end{pmatrix} dx \\ &= 2\pi \int_0^3 x \frac{9x}{\sqrt{x^3+9}}\, dx = \\ &= 18\pi \int_0^3 \frac{x^2}{\sqrt{x^3+9}}\, dx \end{aligned}$$

. 5 2. Set up the integral for the arc length of the portion of the curve $f(x) = x^4$ with $1 \le x \le 2$. You do not need to evaluate the integral.

Solution. The arc length for $f(x) = x^4$ on $[1, 2]$ is given by

$$\int_1^2 \sqrt{1 + [f'(x)]^2}\, dx = \int_1^2 \sqrt{1 + (4x^3)^2}\, dx = \int_1^2 \sqrt{1 + 16x^6}\, dx\ .$$