1. Find the following limit:

\[
\lim_{x \to 1} \frac{\sqrt{x^2 + 8} - 3}{x + 1}
\]

Solution.

\[
\lim_{x \to 1} \frac{\sqrt{x^2 + 8} - 3}{x + 1},
\]

and by multiplying by the conjugate of \(\sqrt{x^2 + 8} - 3\) we get

\[
\lim_{x \to 1} \frac{(\sqrt{x^2 + 8} - 3)(\sqrt{x^2 + 8} + 3)}{(x + 1)(\sqrt{x^2 + 8} + 3)}
\]

which expanded out is

\[
\lim_{x \to 1} \frac{x^2 + 8 - 9}{(x + 1)(\sqrt{x^2 + 8} + 3)} = \lim_{x \to 1} \frac{x^2 - 1}{(x + 1)(\sqrt{x^2 + 8} + 3)}
\]

By simplifying the numerator we get "two perfect squares and a minus sign," so we can factor the numerator to \((x + 1)(x - 1)\).

\[
\lim_{x \to 1} \frac{(x - 1)(x + 1)}{(x + 1)(\sqrt{x^2 + 8} + 3)} \quad \lim_{x \to 1} \frac{x - 1}{\sqrt{x^2 + 8} + 3} = \frac{(-1) - 1}{\sqrt{(-1)^2 + 8} + 3} = \frac{-1}{3},
\]

where we’ve used the limit laws to evaluate the limit.