Math 106, Sect. 350 — Answers to Quiz # 1 — 24 Aug '06

. 6 1. Using a limit of slopes of secant lines, find the slope of $y = x^2 + 3$ at $P = (1, 4)$ and the equation of the tangent line through this point.

Solution. Let Q be the point $(1+h, (1+h)^2+3)$. Notice that $(1+h)^2+3 = 4 + 2h + h^2$. The slope of the line through P and Q is

$$\frac{(4 + 2h + h^2) - 4}{1 + h - 1} = \frac{2h + h^2}{h} = 2 + h.$$

Taking the limit as h approaches 0 gives 2.

So the tangent line has slope 2 and goes through $(1, 4)$. Using the slope-point equation for a line, the tangent line is

$$y - 4 = 2(x - 1).$$

This simplifies to $y = 2x + 2$.

. 4 2. Find the range and domain for $g(t) = \sqrt{-1 + 3^{-t}}$.

Solution. For the domain, we need t so that $-1 + 3^{-t} \geq 0$. Adding 1 to both sides, $3^{-t} \geq 1$. By graphing 3^{-t} or using logarithms, we must have $t \leq 0$ in order to have $3^{-t} \geq 1$. So the domain of g is all numbers less than or equal to 0, i.e., the interval $(-\infty, 0]$.

To find the range, notice that $y = 3^{-t}$, for $t \leq 0$ starts at $(0, 1)$ and, as t decreases, the values increase without bound. Then $y = -1 + 3^{-t}$ is given by translating this graph down by 1. The square root changes the shape, but since $\sqrt{0} = 0$, the range of values is $[0, +\infty)$.