1. For each of the following formulas, say if it is right or wrong, clearly justifying your answer:

(a) \[ \int \tan \theta \sec^2 \theta \, d\theta = \frac{\sec^3 \theta}{3} + C, \]
(b) \[ \int \tan \theta \sec^2 \theta \, d\theta = \frac{1}{2} \tan^2 \theta + C, \]
(c) \[ \int \tan \theta \sec^2 \theta \, d\theta = \frac{1}{2} \sec^2 \theta + C. \]

Solution. The key point is that the formula \( \int f(\theta) \, d\theta = g(\theta) + C \) is right if and only if \( g'(\theta) = f(\theta) \). So, for each formula, we take the derivative of the right hand side and see if it equals \( \tan \theta \sec^2 \theta \).

For the first,
\[
\frac{d}{d\theta} \left( \frac{\sec^3 \theta}{3} \right) = \frac{1}{3} 3 \sec^2 \theta (\sec \theta \tan \theta) = \sec^3 \theta \tan \theta,
\]
which does not equal \( \tan \theta \sec^2 \theta \). (If it did, their ratio would be one, and canceling would show that \( \sec \theta = 1 \) for all \( \theta \).) So the first formula is wrong.

For the second,
\[
\frac{d}{d\theta} \left( \frac{1}{2} \tan^2 \theta \right) = \frac{1}{2} 2 \tan \theta \sec^2 \theta = \tan \theta \sec^2 \theta,
\]
so the second formula is right.

For the third,
\[
\frac{d}{d\theta} \left( \frac{1}{2} \sec^2 \theta \right) = \frac{1}{2} 2 \sec \theta (\sec \theta \tan \theta) = \tan \theta \sec^2 \theta,
\]
so the third formula is also right.

If you are curious why one function could have two different antiderivatives, notice that \( \sec^2 \theta = 1 + \tan^2 \theta \). By changing the arbitrary constant in the second formula to be \( C + 1 \), we can use this trig identity to get the third. The difference between the two formulas is that when we solve for \( C \) using a given initial condition, the constants will be different, depending on which formula we use.