1. If \( x = 2t - t^3 \) and \( y = 1 + t^3 \),

   a. find \( \frac{dy}{dx} \).

   \[
   \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2 - 3t^2} \quad \text{(2 points)}
   \]

   b. find the value(s) of \( t \) for which the slope of the tangent line to the curve is 2.

   \[
   \frac{3t^2}{2 - 3t^2} = 2 \quad \Rightarrow \quad \frac{3t^2}{2 - 3t^2} = 2 \quad \Rightarrow \quad t = \pm \frac{2}{3} \quad \text{(4 points)}
   \]

2. a. Find the linear approximation for \( f(x) = \sqrt[3]{x} \) near 8.

   \[
   L(x) = f(8) + f'(8)(x - 8) \quad \text{(8 points)}
   \]

   \[
   L(x) = 2 + \frac{1}{12} (x - 8) \quad \text{or} \quad L(x) = \frac{4}{3} + \frac{1}{12} x \quad \text{(2 points)}
   \]

   b. Use your approximation to approximate \( \sqrt[3]{7.92} \).

   \[
   L(7.92) = 2 + \frac{1}{12} (7.92 - 8) \quad \text{(plug in: 2 pts)}
   \]

   \[
   = 2 - 0.006 = 1.994 \quad \text{ans: 2 pts}
   \]
3. Given \( f(x) = 3x^4 - 4x^3 - 6x^2 + 6 \),

a. Find and classify the critical x-values of \( f(x) \).

\[ f'(x) = 12x^3 - 12x^2 - 12x = 0 \]

\( x = 0 \) \hspace{1cm} \( x = \frac{1 \pm \sqrt{1 - 4 \cdot (-1)}}{2} = \frac{1 \pm \sqrt{5}}{2} \)

**Critical Points:**
- \( x = 0 \) is a local max.
- \( x = \frac{1 - \sqrt{5}}{2} \) and \( x = \frac{1 + \sqrt{5}}{2} \) are local mins.

b. Find all inflection points.

\[ f''(x) = 36x^2 - 24x - 12 = 0 \]

\( x = -\frac{1}{3} \) or \( 1 \)

**Inflection Points:**
- \( x = -\frac{1}{3} \)
- \( x = 1 \)

4. c. List the interval(s) on which \( f(x) \) is increasing.

\( \left( -\frac{1 - \sqrt{5}}{2}, 0 \right) \cup \left( \frac{1 + \sqrt{5}}{2}, \infty \right) \)

5. d. List the interval(s) on which \( f(x) \) is concave down.

\( \left( -\frac{1}{3}, 1 \right) \)

6. e. Graph \( f(x) \) on the grid below, labeling all intercepts, asymptotes, max/mins, and inflection points on the graph.

\[ y = 3x^4 - 4x^3 - 6x^2 + 6 \]
$y = 3x^4 - 4x^3 - 6x^2 + 6$
4. Find the following limit (be sure to show your work): 
\[ \lim_{x \to \infty} \frac{(\ln x)^2}{x^2} \]
\[ = \lim_{x \to \infty} \frac{\ln x}{x} \cdot \frac{\ln x}{x} \]
\[ = \lim_{x \to \infty} \frac{\ln x}{x} \cdot \lim_{x \to \infty} \frac{\ln x}{x} \]
\[ = \lim_{x \to \infty} \frac{1}{x} \cdot \lim_{x \to \infty} \frac{1}{2x^2} \]
\[ = 0 \]  

if the drop the \( \lim \) early 

5. A rectangle's base, \( b \), is increasing at a rate of 3 cm/min while its height, \( h \), is decreasing at a rate of 1 cm/min. At the time when \( b = 60 \) and \( h = 25 \)

a. how is the area changing?

\[ A = bh \]
\[ \frac{dA}{dt} = b \frac{dh}{dt} + h \frac{db}{dt} \]
\[ = 60(-1) + 25(-5) \]
\[ = 15 \text{ cm}^2/\text{min} \]

b. how is the length of the diagonal changing?

\[ x^2 = b^2 + h^2 \]
\[ x = \sqrt{b^2 + h^2} \]
\[ 60^2 + 25^2 = 65^2 \]
\[ x = 65 \]
6. a. Using Newton's Method, write out the equation for \( x_{n+1} \) when \( f(x) = 3x^2 - \sqrt{10} \).

\[
x_{n+1} = x_n - \frac{3x_n^2 - \sqrt{10}}{6x_n} = \frac{6x_n^2 - 3x_n^2 + \sqrt{10}}{6x_n} = \frac{3x_n^2 + \sqrt{10}}{6x_n}
\]

2. b. \( f(x) \) has a root near \( x = 1 \), find approximations for the values of \( x_1 \) and \( x_2 \) to 6 decimal places. You do not need to show your work.

\[
\begin{align*}
x_1 &= 1.027046 \\
x_2 &= 1.026690
\end{align*}
\]

7. Given \( f(x) = \sin x \) defined on the interval \([-\frac{\pi}{2}, \frac{\pi}{2}]\):

a. Verify the hypotheses of the Mean Value Theorem for \( f(x) \).

\[
\begin{align*}
&\text{1. } f(x) \text{ is continuous on } [-\frac{\pi}{2}, \frac{\pi}{2}] \text{ (since } \sin x \text{ is everywhere continuous)} \\
&\text{2. } f(x) \text{ is differentiable on } (-\frac{\pi}{2}, \frac{\pi}{2}) \text{ (since } \cos x \text{ is derivable there and has no corners or cusps)}
\end{align*}
\]

b. Find the value or values of \( c \) that satisfy the equation \( \frac{f(b) - f(a)}{b - a} = f'(c) \) in the conclusion of the Mean Value Theorem.

\[
\begin{align*}
\frac{\sin(\frac{\pi}{2}) - \sin(-\frac{\pi}{2})}{\frac{\pi}{2} - (-\frac{\pi}{2})} &= \cos c \text{ where } c \in (-\frac{\pi}{2}, \frac{\pi}{2}) \\
\frac{2}{\pi} &= \cos c \\
\frac{2}{11} &= \cos c
\end{align*}
\]