1. Find \( f'(x) \) for the following functions. (You need not simplify your answers.)

   (a) \( f(x) = 8x^5 - 2x^3 + \frac{3}{x} \)

   (b) \( f(x) = 2 + 2\sqrt{x} - \frac{1}{\sqrt{x}} \)

   (c) \( f(x) = \frac{x^2 - 3x + 1}{x} \)

   \textit{Solution.} For part (a), we use the power rule with \( n = 5 \), \( n = 3 \), and \( n = -1 \).

   \[ f'(x) = 8 \cdot 5x^4 - 2 \cdot 3x^2 + 3\left(-1\right)x^{-2} = 40x^4 - 6x^2 - \frac{3}{x^2}. \]

   For part (b), notice \( f(x) = 2 + 2x^{1/2} - x^{-1/2} \) and so, using the power rule with \( n = 0 \), \( n = 1/2 \), and \( n = -1/2 \), we have

   \[ f'(x) = 0 + 2 \cdot \frac{1}{2}x^{-1/2} - \frac{-1}{2}x^{-3/2} = \frac{1}{\sqrt{x}} + \frac{1}{2x^{3/2}}. \]

   For part (c), we rewrite \( f(x) \) using

   \[ f(x) = \frac{x^2}{x} - \frac{3x}{x} + \frac{1}{x} = x - 3 + x^{-1} \]

   and then use the power rule to get

   \[ f'(x) = 1 + 0 - x^{-2}. \]

2. Evaluate the following limits:

   (a) \( \lim_{x \to +\infty} 5 + \frac{x^2 - 16}{5x^2 - 20} \)

   (b) \( \lim_{x \to 2^+} \frac{2x - 5}{x^2 - 3x + 2} \)

   (c) \( \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \)
Solution. For part (a),

\[
\lim_{x \to +\infty} 5 + \frac{x^2 - 16}{5x^2 - 20} = \lim_{x \to +\infty} 5 + \lim_{x \to +\infty} \frac{x^2 - 16}{5x^2 - 20} = 5 + \lim_{x \to +\infty} \frac{x^2 - 16}{5x^2 - 20} = 5 + \lim_{x \to +\infty} \frac{1 - \frac{16}{x^2}}{\frac{5}{x^2} - \frac{20}{x^2}} = 5 + \frac{1}{5},
\]

and so the limit is \(\frac{26}{5} = 5.2\).

For part (b), substituting \(x = 2\) into the numerator and denominator gives \(-1\) and \(0\), respectively. Since we’re dividing a nonzero number by zero, there is a vertical asymptote and the limit is either \(+\infty\) or \(-\infty\). Factoring the denominator, we have

\[
\lim_{x \to 2^+} \frac{2x - 5}{x^2 - 3x + 2} = \lim_{x \to 2^+} \frac{2x - 5}{(x - 1)(x - 2)}
\]

As \(x\) approaches \(2\) from above, \(2x - 5\) approaches \(-1\), \(x - 1\) approaches \(1\) and \(x - 2\) approaches \(0\) from above (is positive). Since we have two positive quantities and one negative, the limit is \(-\infty\).

For part (c),

\[
\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = \lim_{x \to 9} \frac{\sqrt{x} - 3}{(\sqrt{x} + 3)(\sqrt{x} - 3)} \lim_{x \to 9} \frac{1}{\sqrt{x} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{6}.
\]

3. Find the horizontal and vertical asymptotes for

\[
f(x) = \frac{2x^2 + 8}{x^2 + 3x - 10}.
\]

As always, explain your reasoning.

Solution. To find the vertical asymptotes, we look for values of \(x\) where the denominator is zero. If the numerator is not zero, then there is a vertical asymptote and if the numerator is zero, we need to take limits to figure out what is happening.

So, we solve \(x^2 + 3x - 10 = 0\), which factors as \((x + 5)(x - 2) = 0\), giving \(x = -5\) and \(x = 2\) as possible asymptotes. Since \(2x^2 + 8\) is not zero for \(x = -5\) or \(x = 2\), these are both vertical asymptotes.
To find the horizontal asymptotes, we compute
\[
\lim_{x \to \infty} \frac{2x^2 + 8}{x^2 + 3x - 10} = \lim_{x \to \infty} \frac{\frac{2x^2}{x^2} + \frac{8}{x^2}}{\frac{x^2}{x^2} + \frac{3x}{x^2} - \frac{10}{x^2}} = \lim_{x \to \infty} \frac{2 + \frac{8}{x^2}}{1 + \frac{3}{x} - \frac{10}{x^2}} = 2
\]
So there is a horizontal asymptote, \( y = 2 \).

4. Find the equation of the tangent line to the graph of \( y = f(x) = 4/x + 5x \) when \( x = 2 \).

\[\text{Solution.}\]
To find the equation of the tangent line, we need to find \( f(2) \), as the line goes through the point \((2, f(2))\), and we need to find \( f'(2) \), since this is the slope of the tangent line at \( x = 2 \).
First, \( f(2) = 4/2 + 5 \cdot 2 = 2 + 10 = 12 \), and second, rewrite \( f(x) \) as \( f(x) = 4x^{-1} + 5x \), so
\[f'(x) = -4x^{-2} + 5 = \frac{-4}{x^2} + 5.
\]
Thus, \( f'(2) = -4/2^2 + 5 = -1 + 5 = 4 \). So the equation of the line is
\[y - 12 = 4(x - 2).
\]

5. Let \( f(x) = \begin{cases} \frac{2x^2 + x - 20}{x - 4} & \text{if } x < 4, \\ 5x - 4 & \text{if } x \geq 4. \end{cases} \)

(a) Find \( \lim_{x \to 4^+} f(x) \) and \( \lim_{x \to 4^-} f(x) \).

(b) Is \( f(x) \) continuous at 4? Why or why not?

\[\text{Solution.}\]
For part a), we have
\[\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} 5x - 4 = 5(4) - 4 = 16\]
and
\[\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{2x^2 + x - 20}{x - 4}\]
Substituting \( x = 4 \) in the fraction makes the denominator zero and the numerator \( 2 \cdot 4^2 + 4 - 20 = 16 \). Since the numerator is not zero, this limit is either \( +\infty \) or \( -\infty \). In fact, since the numerator is positive and denominator is negative (as \( x \) is a bit less than 4), the limit is \( -\infty \).
For part b), since
\[\lim_{x \to 4^-} f(x) \neq \lim_{x \to 4^+} f(x),\]
we have \( \lim_{x \to 4} f(x) \) does not exist and so the function cannot be continuous at 4.
6. Suppose the cost of producing \( x \) calculus textbooks is given by \( C(x) = 50 + x - \frac{x^2}{100} \), in hundreds of dollars.

(a) What is the **average cost** of a book if 150 books are made?
(b) What is the **marginal cost** function?
(c) What is the **marginal cost** when \( x = 20 \)?
(d) What is the exact cost of the 21st book?
(e) If the book sells for $95 each, what are the **revenue** function \( R(x) \) and the **profit** function \( P(x) \), in hundreds of dollars?

**Solution.** For part a), there are two acceptable answers. Using the usual meaning of average cost, it is

\[
\frac{C(150)}{100} = \frac{50 + 150 - \frac{150^2}{100}}{100} = \frac{-25}{100} = -0.25
\]

which means the cost per book is -$25 (since units are hundreds of dollars). Using the average rate of change from \( x = 0 \) to \( x = 150 \), we have

\[
\frac{C(150) - C(0)}{100 - 0} = \frac{-25 - 100}{100} = -1.25
\]

which means the average rate of change of the cost in making the first hundred and fifty books is -$125 per book.

For part b), the marginal cost function is

\[
C'(x) = 1 - \frac{2x}{100} = 1 - \frac{x}{50}.
\]

For part c), we have

\[
C'(20) = 1 - \frac{20}{50} = 0.6
\]

which, with units of hundreds of dollars, is $60.

For part d), the exact cost is the cost of making 21 books minus the cost of making the first 20, i.e.,

\[
C(21) - C(20) = \left( 50 + 21 - \frac{21^2}{100} \right) - \left( 50 + 20 - \frac{20^2}{100} \right) = 1 - \frac{41}{100} = 0.59
\]

which, with units of hundreds of dollars, is $59.

For part e), the revenue is \( R(x) = 0.95x \) and the profit is

\[
P(x) = R(x) - C(x) = 0.95x - \left( 50 + x - \frac{x^2}{100} \right) = -50 - 0.05x + \frac{x^2}{100}.
\]
7. Using the definition, find the derivative of the function \( f(x) = 2x^2 - 7x - 5 \) when \( x = 3 \).

**Solution.** By the definition,

\[
f'(3) = \lim_{h \to 0} \frac{f(3 + h) - f(3)}{h}
\]

Now,

\[
f(2 + h) = 2(3 + h)^2 - 7(3 + h) - 5
= 2(9 + 6h + h^2) - 21 - 7h - 5
= 18 + 12h + 2h^2 - 21 - 7h - 5 = -8 + 5h + 2h^2
\]

while \( f(3) = 2(3)^2 - 7 \cdot 3 - 5 = 18 - 21 + 5 = -8 \). Thus,

\[
f'(3) = \lim_{h \to 0} \frac{f(3 + h) - f(3)}{h}
= \lim_{h \to 0} \frac{-8 + 5h + 2h^2 - (-8)}{h}
= \lim_{h \to 0} \frac{5h + 2h^2}{h}
= \lim_{h \to 0} 5 + 2h = 5
\]

So \( f'(3) = 5 \).

As a check, we compute \( f'(x) = 4x - 7 \) according to the rules, and so \( f'(3) = 4 \cdot 3 - 7 = 5 \), so the two ways of computing \( f'(3) \) agree.