MATH 104 HOUR 3 REVIEW QUESTIONS

1. Suppose the cost and revenue functions are given by $C = 10,000 + 48.9x$ and $R = 68.5x$.
(a) Find the average cost function.
(b) What is the limit of the average cost as x approaches infinity?
(c) Find the average profit when x is 1 million, 2 million, and 10 million.

2. Find $g''(2)$ if $g(x) = \frac{3x+1}{5x-7}$.

3.(a) Analyze the graph of $f(x) = \frac{x}{x^2+4}$, giving the intervals where f is increasing, decreasing, relative extrema, intervals of concavity, asymptotes, intercepts, and sketch a graph on a set of axes.

(b) Do the same for $f(x) = \frac{x}{x^2-4}$.

4. Use calculus methods to find the absolute maximum value M and the absolute minimum value m of the functions:
(a) $f(x) = 8 + 3x^2 - x^3$ on the closed interval $[1, 3]$.

(b) $f(x) = 3x^4 - 6x^2 + 2$ on the closed interval $[0, 2]$.

(c) $f(x) = \frac{2x}{x^2+1}$ on $[-1, 2]$.

5. Graph and analyze the function $f(x) = x^2 e^{-x}$, including relative extrema, concavity, and asymptotes.

6. (a) Find $g''(e)$ if $g(x) = x^2 \ln(x)$.

(b) Find all critical numbers of the function $f(x) = x^2 e^{-5x}$.

7. Sketch a graph of the following functions, and be sure to label any extrema and points of infection:
(a) $y = -x^3 + 3x^2 + 9x - 2$

(b) $y = x^5 - 5x$

8. Find all <u>points of inflection</u> of $y = f(x) = x^4 - 4x^3 + 116$.

9. Find the points of inflection of the graphs of $f(x) = x(6 - x)^2$.

10. Given the cost function $C = C(x) = 2x^2 + 15x + 800$ dollars, use <u>calculus methods</u> to determine the number of units x that should be produced in order to <u>minimize</u> the <u>average cost</u> per unit.

11. Let $y = f(x)$ be a function such that $f'(x) = x^3(x - 1)^2(x + 4)(x - 6)$ for all $x \in (-\infty, \infty)$. List the critical numbers, the open intervals on which f is increasing, and the number(s) at which f has a relative maximum.

12. (a) Use calculus-based procedures to find two positive numbers x and y such that $xy = 100$ and the function $S = x + 4y$ is a minimum.

(b) Find two numbers whose difference is 50 and whose product is a minimum.

13. Find the points of inflection of the graph of $f(x) = (x-2)^3(x-1)$.

14. Find an equation of the tangent line to the graph of $y = f(x) = e^{4x-3} - \ln(x^4)$ at the point $(1, e)$.

15. Suppose that a manufacturer can sell x widgets at a price of $80 - .02x$ dollars each and assume that it costs $40x + 1500$ dollars to produce all x of them.
(a) Find the revenue function, and the profit function.

(b) Determine the value of x which will maximize the revenue function.

(c) Determine the value of x that will maximize the profit function.

16. Let $y = f(x)$ be a function such that $f''(x) = x^2(x+4)(x-2)$ for all $x \epsilon (-\infty, +\infty)$.

(a) List the open interval(s) where the graph of f is concave up.

(b) List the number(s) x where $(x, f(x))$ is a point of inflection on the graph of f.

17. Find an equation of the tangent line to the graph of the curve $y = f(x) = \ln(x^3) - 6x^2$ at the point $(1, -6)$.

18. Find an equation of the tangent line to the graph of the curve $y = \frac{\ln(x)}{x}$ at the point $(e^2, 2e^{-2})$.

19. Determine the relative extrema of the function $f(x) = x^2 - 3\ln(x)$.

20. Sketch the graph of the following functions, showing all relative extrema, points of inflection, intercepts, and asymptotes. Also, state the domain of each.

(a) $y = \frac{2}{x-3}$

(b) $y = \frac{2x}{x^2-1}$.

21. Suppose that a manufacturer wants to make an open box with a square base which is to hold 25 liters. The material used is plastic and the material for the bottom will cost three times as much as the material for the sides. What are the dimensions which will minimize the total cost?