1. Find the absolute minimum of \( f(x) = 2x^2 - 12x + 18 \) on \([2, 6]\) and the \(x\)-value where it occurs.

   Solution. First, \( f'(x) = 4x - 12 \) and so \( f'(x) = 0 \) when \( x = 3 \). Notice that \( f'(x) \) is never undefined.

   To see if this is a minimum, we must evaluate \( f(x) \) at \( x = 3 \) and the endpoints:
   \( f(2) = 8 - 24 + 18 = 2 \), \( f(3) = 18 - 36 + 18 = 0 \), and \( f(6) = 72 - 72 + 18 = 18 \).

   Thus, the minimum is 0 at \( x = 3 \).

2. Find positive numbers \( x \) and \( y \) with \( x + y = 15 \) so that \( x^2 y \) is as large as possible.

   Solution. First, notice that \( y = 15 - x \) and the function to be maximized is

   \[ f(x) = x^2(15 - x) = 15x^2 - x^3. \]

   Thus, \( f'(x) = 30x - 3x^2 = 3x(10 - x) \). So the critical numbers are \( x = 0 \) and \( x = 10 \).

   To see where we have an absolute max, we can chart \( f' \). Using \( f'(1) = 3 \cdot 9 > 0 \), we have that \( f \) increases on \((0, 10)\). Using \( f'(11) = 33 \cdot (-1) < 0 \), we have the \( f \) decreases on \((10, +\infty)\). Thus \( f \) has a relative max at \( x = 10 \) and because we are only interested in positive \( x \), this is an absolute max.

   The other way to see that the absolute max is at \( x = 10 \) is to notice that for \( y = 15 - x \) to be positive, \( x \) must be at most 15. Thus, we are looking for the absolute max on the interval \([0, 15]\). Trying the endpoints and the critical numbers, we have \( f(0) = 0 \), \( f(10) = 1500 - 1000 = 500 \), \( f(15) = 0 \). So the absolute max is at \( x = 10 \).

   Either way, the choice of numbers to maximize \( x^2 y \) is \( x = 10 \) and \( y = 15 - 10 = 5 \).