**Math 104-004** **Answers to Quiz # 8p** **2 Nov 07**

. 4 1. Find the absolute minimum of $f(x) = 2x^2 - 12x + 18$ on $[2, 6]$ and the $x$-value where it occurs.

*Solution.* First, $f'(x) = 4x - 12$ and so $f'(x) = 0$ when $x = 3$. Notice that $f'(x)$ is never undefined.

To see if this is a minimum, we must evaluate $f(x)$ at $x = 3$ and the endpoints: $f(2) = 8 - 24 + 18 = 2$, $f(3) = 18 - 36 + 18 = 0$, and $f(6) = 72 - 72 + 18 = 18$. Thus, the minimum is 0 at $x = 3$.

. 6 2. Find positive numbers $x$ and $y$ with $x + y = 15$ so that $x^2y$ is as large as possible.

*Solution.* First, notice that $y = 15 - x$ and the function to be maximized is

$$f(x) = x^2(15 - x) = 15x^2 - x^3.$$

Thus, $f'(x) = 30x - 3x^2 = 3x(10 - x)$. So the critical numbers are $x = 0$ and $x = 10$.

To see where we have an absolute max, we can chart $f'$. Using $f'(1) = 3 \cdot 9 > 0$, we have that $f$ increases on $(0, 10)$. Using $f'(11) = 33 \cdot (-1) < 0$, we have the $f$ decreases on $(10, +\infty)$. Thus $f$ has a relative max at $x = 10$ and because we are only interested in positive $x$, this is an absolute max.

The other way to see that the absolute max is at $x = 10$ is to notice that for $y = 15 - x$ to be positive, $x$ must be at most 15. Thus, we are looking for the absolute max on the interval $[0, 15]$. Trying the endpoints and the critical numbers, we have $f(0) = 0$, $f(10) = 1500 - 1000 = 500$, $f(15) = 0$. So the absolute max is at $x = 10$.

Either way, the choice of numbers to maximize $x^2y$ is $x = 10$ and $y = 15 - 10 = 5$.