Math 104-004 **Answers to Quiz # 7p** **26 Oct 07**

. 6 1. Find all relative extrema of $f(x) = x^3 + 3x^2 - 24x + 2$ and the x-value where each occurs.

Solution. First, $f'(x) = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8) = 3(x + 4)(x - 2)$. Thus, $f'(x) = 0$ when $x = -4$ or when $x = 2$. Notice that $f'(x)$ is never undefined.

Charting $f'(x)$, we have $f'(-5) = 3(-1)(-7) > 0$, $f'(0) = -24 < 0$, and $f'(3) = 3(7)(1) > 0$. Since there is a sign change of $f'(x)$ at both $x = -4$ and at $x = 2$, both of these x-values give relative extrema.

As $f(-4) = (-4)^3 + 3(-4)^2 0 - 24(-4) + 2 = 82$, we have a relative extrema of 82 at $x = -4$. As $f(2) = (2)^3 + 3(2)^2 - 24(2) + 2 = -26$, we have a relative extrema of -26 at $x = 1$.

. 4 2. For the function $f(x) = x^3 + 3x^2 + 3x + 4$, find its inflection points, the open intervals where it is concave up, and the open intervals where it is concave down.

Solution. First, $f'(x) = 3x^2 + 6x + 3$ and so $f''(x) = 6x + 6$. Thus, $f''(x) = 0$ when $x = -1$.

Charting $f''(x)$, we have $f''(-2) = 6(-2) + 6 < 0$ and $f''(0) = 6 > 0$, so $f''(x)$ changes sign at $x = -1$. As $f(-1) = -1 + 3 - 3 + 4 = 3$, we have that $(-1, 3)$ is an inflection point. Since $f''(x)$ is negative on $(-\infty, -1)$, the graph is concave down on that interval. Since $f''(x)$ is positive on $(-1, +\infty)$, the graph is concave up on that interval.