1. Find the derivative of \( h(x) = \ln(\sqrt{e^{5x} + 4x}) \).

**Hint:** Simplify the function before differentiating.

**Solution.** This question is similar to questions 8 and 10 on page 260.

First, rewrite \( h(x) \) as

\[
h(x) = \ln\left(\sqrt{e^{5x} + 4x}\right) = \frac{1}{2} \ln(e^{5x} + 4x)
\]

using the property \( \ln(a^r) = r \ln(a) \).

Using the rule for the derivative of \( \ln(f(x)) \) and then the rule for the derivative of \( e^{f(x)} \), we get

\[
h'(x) = \frac{1}{2} \cdot \frac{e^{5x} + 4}{(e^{5x} + 4x)^{1/2}}
\]

If you don’t simplify the function first, then using the rule of the derivative of \( \ln(f(x)) \) and then the chain rule, and then the rule for the derivative of \( e^{f(x)} \), we get

\[
h'(x) = \frac{1}{2} \cdot \frac{e^{5x} + 4}{(e^{5x} + 4x)^{1/2}} \cdot (e^{5x} + 4)
\]

\[
= \frac{5e^{5x} + 4}{2(e^{5x} + 4x)^{1/2} \cdot (e^{5x} + 4x)^{1/2}}
\]

\[
= \frac{5e^{5x} + 4}{2(e^{5x} + 4x)}.
\]

2. For the function \( f(x) = x^3 + 6x^2 - 36x + 2 \), find its critical numbers, the open intervals where it is increasing, and the open intervals where it is decreasing.

**Solution.** The derivative of \( f(x) \) is \( f'(x) = 3x^2 + 12x - 36 = 3(x^2 + 4x - 12) = 3(x + 6)(x - 2) \). Thus, the critical numbers of \( f(x) \) are \(-6\) and \(2\).

By drawing the number line and choosing points in each of the intervals \((-\infty, -6)\), \((-6, 2)\) and \((2, +\infty)\), we can see if the function is increasing or decreasing.

For \((-\infty, -6)\), we pick \( x = -7 \) and get \( f'(-7) = 3(-7+6)(-7-2) = 3(-1)(-9) > 0 \). For \((-6, 2)\), we pick \( x = 0 \) and get \( f'(0) = 3(6)(-2) < 0 \). For \((2, +\infty)\), we pick \( x = 3 \) and \( f'(4) = 3(3+5)(3-2) = 3 \cdot 8 \cdot 1 > 0 \).

By the test for increasing/decreasing, \( f(x) \) is increasing on \((-\infty, -6)\) and \((2, +\infty)\), while it is decreasing on \((-6, 2)\).