**Hibi Rings in Representation Theory**

This talk will attempt to sketch some uses of Hibi rings in representation theory.

Let $X$ be a partially ordered set (poset). The set $(\mathbf{R}^+)^X_\geq$ of order preserving non-negative functions on $X$ is an open convex cone in the vector space $\mathbf{R}^X$ of all real-valued functions on $X$. The subset $(\mathbf{Z}^+)^X_\geq = (\mathbf{R}^+)^X_\geq \cap \mathbf{Z}^X$ of non-negative integer-valued order-preserving functions is the intersection of $(\mathbf{R}^+)^X_\geq$ with the integer lattice in $\mathbf{R}^X$. It is a finitely generated semigroup. The semigroup ring $\mathbf{C}((\mathbf{Z}^+)^X_\geq)$ is called a *Hibi ring.*

If $X$ has the trivial order – no two elements are comparable – then $(\mathbf{Z}^+)^X_\geq = (\mathbf{Z}^+)^X$ is simply the free semigroup on $X$, and the associated Hibi ring is just a polynomial ring. Hibi rings form a large yet relatively tractable and explicitly describable class of semigroup rings. The algebraic varieties associated to semigroup rings are affine toric varieties.

Around 1940, Hodge gave a description of a basis for the coördinate ring of the (full) flag manifold for $GL_n$, aka the *flag algebra.* Since the flag algebra for $GL_n$ contains one copy of each irreducible representation of $GL_n$, this also provides a description of a basis for the irreducible representations of $GL_n$.

Hodge's result became known as *standard monomial theory.* In the decades following, efforts to understand the principles behind standard monomial theory led to substantial activity in commutative algebra, and provided part of the impetus for the development of combinatorial algebra. However, it was only after the work of Gonciulea and Lakshmibai (in the late 1990s) identified a connection between Hodge's work and an alternate description of bases for irreducible representations due to Gelfand and Tsetlin (1949) that a truly simple picture emerged: the coördinate ring of the flag variety for $GL_n$ looks a lot like (technically, is a flat deformation of) the Hibi ring on a certain simple poset, known as the Gelfand-Tsetlin poset. Recently, it has been found that several other algebras arising naturally in representation theory can be described using Hibi rings. Moreover, the Hibi ring perspective has shed new light on other basic questions, such as Littlewood-Richardson coefficients (that describe the decomposition of tensor products of representations of $GL_n$).