**Cubic Column Relations in Truncated Moment Problems**

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**Abstract**. Inverse problems naturally occur in many branches of science and mathematics. An inverse problem entails finding the values of one or more parameters using the values obtained from observed data. A typical example of an inverse problem is the inversion of the Radon transform. Here a function (for example of two variables) is deduced from its integrals along all possible lines. This problem is intimately connected with image reconstruction for X-ray computerized tomography.

Moment problems are a special class of inverse problems. While the classical theory of moments dates back to the beginning of the 20th century, the systematic study of *truncated* moment problems began only a few years ago. In this talk we will first survey the elementary theory of truncated moment problems, and then focus on those problems with cubic column relations.

For a degree $2n$ real $d$-dimensional multisequence $\beta \equiv \beta^{(2n)} = \{\beta_i\}_{i \in Z_+^d, |i| \leq 2n}$ to have a representing measure $\mu$, it is necessary for the *associated moment matrix* $M(n)$ to be positive semidefinite, and for the *algebraic variety* associated to $\beta$, $V_\beta$, to satisfy rank $M(n) \leq$ card $V_\beta$ as well as the following *consistency condition*: if a polynomial $p(x) \equiv \sum_{|i| \leq 2n} a_i x^i$ vanishes on $V_\beta$, then $p(\beta) := \sum_{|i| \leq 2n} a_i \beta_i = 0$. In previous joint work with L. Fialkow and M. Möller, we proved that for the *extremal* case (rank $M(n) =$ card $V_\beta$), positivity and consistency are sufficient for the existence of a (unique, rank $M(n)$-atomic) representing measure.

In recent joint work with Seonguk Yoo we consider cubic column relations in $M(3)$ of the form (in complex notation) $Z^3 = itZ + u\bar{Z}$, where $u$ and $t$ are real numbers. For $(u, t)$ in the interior of a real cone, we prove that the algebraic variety $V_\beta$ consists of exactly 7 points, and we then apply the above mentioned solution of the extremal moment problem to obtain a necessary and sufficient condition for the existence of a representing measure. This requires a new representation theorem for sextic polynomials in $Z$ and $\bar{Z}$ which vanish in the 7-point set $V_\beta$. Our proof of this representation theorem relies on two successive applications of the Fundamental Theorem of Linear Algebra.