Question 1. How many digits does the number $25^88^7$ have?

(A) 17    (B) 18    (C) 19    (D) 20    (E) 21

Answer. $8^2 = 4^2 = 2^4 = 2^2$, and $25^4 = 10^5 = 10^3$, so $25^88^7 = 10^{15} \cdot 2^3 = 32 \cdot 10^6$, which is a 32 with 16 zeros after it, so has 18 digits.

Question 2. In a class of 30 students, 20 students study history, 16 students study chemistry, and 3 students study neither. How many students study both?

(A) 4    (B) 6    (C) 8    (D) 9    (E) 12

Answer. 27 students study at least one subject, and if $x$ represents those studying both, then $20 + 16 - 36 + x = 27$, so $x = 36 - 27 = 9$.

Question 3. If $a + b = 2$ and $a^2 + b^3 = 3$, then what is $a + b^3$?

(A) 4    (B) 5    (C) 6    (D) 7    (E) 8

Answer. $6 = (a + b)(a^2 + b^3) = (a^3 + b^3) + b(a^2 + b^3) = (a^3 + b^3) + ab(a + b) = (a^3 + b^3) + 2ab$, and $2ab = (a + b)^2 - (a^2 + b^3) = 2^2 - 3 = 1$, so $a^3 + b^3 = 6 - 1 = 5$.

Question 4. A 24 inch piece of string is folded in thirds, and then all three strands are cut 3/5th of the way from one end to the other. What is the length, in inches, of the longest piece of string that results?

(A) $\frac{24}{5}$    (B) $\frac{28}{5}$    (C) $\frac{32}{5}$    (D) $\frac{42}{5}$    (E) 48

Answer. Folded in thirds the string appears 8 inches long. When cut, the pieces are 3/5, then 2 \cdot \frac{3}{5}, then 2 \cdot \frac{3}{5}, then 2\cdot \frac{3}{5} of this length. The longest is $8 \cdot \frac{3}{5} = \frac{48}{5}$.

Question 5. Circles of radius 3 and 5 are lying on a horizontal line so that they touch (see figure). A line is extended through the centers of the circles. What is the distance from the point $A$ to the point $B^*$?

(A) 10    (B) $\sqrt{129}$    (C) $\sqrt{229}$    (D) $\sqrt{135}$    (E) 12

Answer. The line segment joining the centers has length 8, and using similar triangles the segment from A to the center of the smaller circle has length satisfying $h/3 = (h + 8)/5$, so $h = 12$. Then the smaller right triangle shows that the length of $AB$ satisfies $x^2 + 3^2 = 12^2$, so $x = \sqrt{135}$.

Question 6. $\sqrt{213}$ is closest to which of the following numbers?

(A) 13    (B) 14    (C) 15    (D) 16    (E) 17

Answer. $225 < 243 < 256$, so $15 < \sqrt{213} < 16$. $(31/2)^2 = 961/4 = 240 + 1/4 < 243$, so $31/2 < \sqrt{213}$ and so $\sqrt{213}$ is closest to 16.

Question 7. What is the area of the region in the $x$-$y$ plane satisfying the inequalities $xy \leq 0$ and $x + 1 \leq y \leq x + 2$?

(A) $1/2$    (B) $\pi/4$    (C) $\pi/2$    (D) 2    (E) 5/2

Answer. $xy \leq 0$ means that the region lies in the 2nd and 4th quadrants. The lines $y = x + 1$ and $y = x + 2$ do not meet the 4th quadrant. In the 2nd quadrant they cut off right triangles with areas $\frac{1}{2} \cdot 1 = 1/2$ and $\frac{1}{2} \cdot 2 = 1$, so the region we want has area $2 - 1/2 = 3/2$.

Question 8. The set $\{1, 2, 3\}$ has 8 = $2^3$ subsets: $\{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}, \emptyset\}$. The set $\{8\} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ has 256 = $2^8$ subsets. If you add the elements of each of the 256 subsets of $\{8\}$ together, and then add those 256 sums together, what will the total be?

(A) 2048    (B) 3072    (C) 3600    (D) 4608    (E) 4800

Answer. Each of the numbers $a$ will appear in half of the subsets, since we can pair a subset not containing $a$ with the subset obtained by including $a$ in the set. So each number will appear 128 times. Summing the sums amounts to adding $128 \times 1 \div 2 \div 2$’s, and so on, yielding $128(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) = 128(36) = 512 \times 9 = 4500 + 108 = 4608$.

Question 9. The area of the largest triangle that can be inscribed in a semi-circle of radius $r$ is

(A) $\pi r^2$    (B) $2 \pi r$    (C) $2 \pi^2$    (D) $\frac{1}{2} \pi^2$    (E) None of these.

Answer. Thinking of the diameter as lying on a horizontal line, any triangle we draw will have larger base and altitude if we replace two corners with the ends of the diameter. It remains to determine where the third point should go, but since all of these triangles have the same base, we want the largest altitude, which is at the center of the circle. This triangle has area $\frac{1}{2}(2r)(r) = \pi r^2$.

Question 10. Among all ordered pairs of real numbers $(x, y)$ which satisfy $x^2 + y^2 = x + y$, what is the largest valued $x$?

(A) $\frac{2 - \sqrt{2}}{2}$    (B) $\frac{1 + \sqrt{2}}{2}$    (C) $\frac{1 + \sqrt{3}}{2}$    (D) $\frac{2 + \sqrt{3}}{2}$    (E) $\frac{2 - \sqrt{3}}{2}$

Answer. We need $x^2 - x + y^2 - y = 0$; completing the squares, this is $(x - 1/2)^2 + (y - 1/2)^2 = 1/2$. This is a circle of radius $1/\sqrt{2}$ centered at $(1/2, 1/2)$; the point furthest to the right has x-coordinate $1/2 + 1/\sqrt{2} = (\sqrt{2} + 1)/2$.

Question 11. If $f(x) = 5 - 2x$, then what is $f(f(f(3)))$ ?

(A) $-9$    (B) $-3$    (C) $-1$    (D) 3    (E) 7

Answer. $f(3) = 5 - 2(3) = -1$, $f(-1) = 5 - 2(-1) = 7$, and $f(7) = 5 - 2(7) = -9$.

Question 12. A dog is tied to the corner of a 10 foot building by a 12 foot leash. What is the area (in square feet) of the region that the dog can reach?

(A) $88\pi$    (B) $99\pi$    (C) $105\pi$    (D) $109\pi$    (E) $115\pi$

Answer. The leash reaches 12 feet down the long side, through 270 degrees to the end of the short side, and then through 90 degrees a further 2 feet around the corner of the short side. This encompasses an area of $(3/4)(12)^2 + (1/4)(2)^2 = 108\pi + \pi = 109\pi$.

Question 13. Two circles of radius 3 each pass through the center of the other (see figure). What is the area of the region that lies inside of both circles?

(A) $6\pi - 9\sqrt{3}/2$    (B) $4\pi + 3\sqrt{3}/2$    (C) $9\pi - 3\sqrt{3}$    (D) $8\pi - 3\sqrt{3}/2$    (E) $12\pi + 4\sqrt{3}$

Answer. All of the line segments are radii, so have length 3. The region is covered by two sectors, one for each circle, having an angle of 120 degrees, and area $(1/3)\pi \cdot 3^2 = 3\pi$. They overlap on the two triangles, which have total area $(1/2)(3)(3\sqrt{3}/2) = 9\sqrt{3}/2$. So the total area is $2(3\pi) - 9\sqrt{3}/2 = 6\pi - 9\sqrt{3}/2$.

Question 14. The values of $x$ which satisfy both $|x - 4| > 5$ and $|x - 5| < 6$ are precisely the values of $x$ which satisfy $a < x < b$. What is $a + b$?

(A) 2    (B) 8    (C) 10    (D) 20    (E) None of these.
Answer. The first inequality requires $x - 4 > 5$ or $4 - x > 5$, so $x > 9$ or $x < -1$. The second requires $x - 5 < 6$ and $5 - x < 6$, so $x < 11$ and $-1 < x$. To satisfy both we need $x > 9$ and $x < 11$, so $9 < x < 11.$

**Question 15.** For how many integers $n$ is $\frac{5n + 26}{2n + 3}$ also an integer?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

**Answer.** $\frac{5n + 26}{2n + 3} = 2 + \frac{n + 20}{2n + 3} = 3 - \frac{17}{2n + 3}$ both $n + 20$ and $\frac{n + 17}{2n + 3}$ must be integers, as must their difference, $\frac{37}{2n + 3}$ but since 37 is prime, these $2n + 3$ must be $\pm 1$ or $\pm 37$, which occurs for $n = -1, 2, 17, -20.$

**Question 16.** Find all integer solutions to $3^x - 2x = 8.$

(A) There are no integer solutions (B) $x = -2, 4$ (C) $x = -3, 2$ (D) $x = 4$ (E) $x = -3, -2$

**Answer.** Since the only integer powers of 3 and 7 that are equal are $3^0 = 1 = 7^0$, we need $x^2 - 2x - 8 = 0$ and $x^2 - x - 12 = 0$, that is, $(x - 4)(x + 2) = 0$ and $(x - 4)(x + 3) = 0$. The only common solution is $x = 4$.

**Question 17.** It takes Sue 90 minutes to mow her mother’s yard. Her sister Tawnya can do it in 60 minutes. How long would it take them to mow the yard if they worked together using two mowers?

(A) 36 minutes (B) 40 minutes (C) 42 minutes (D) 75 minutes (E) 150 minutes

**Answer.** Sue can mow $\frac{1}{90}$ of the yard per minute, and Tawnya can mow $\frac{1}{60}$ of the yard per minute, so together they can mow $\frac{1}{90} + \frac{1}{60} = \frac{2}{150} + \frac{5}{150} = \frac{7}{150}$ of the yard per minute, so they can finish in 21.43 minutes, which is not an option.

**Question 18.** A box contains 3 red and 7 blue balls. If two balls are removed at random, what is the probability that they have the same color?

(A) 2/5 (B) 7/15 (C) 1/2 (D) 8/15 (E) 3/5

**Answer.** There are 10 balls, so there are 10 * 9 = 90 ways to remove two, one after the other. For 3 * 2 = 6 of these, both are red; for 7 * 6 = 42 of these, both are blue. So 48 times out of 90, or 8/15 of the time, they are the same color.

**Question 19.** Given the three equations below, for what value of $k$ will their respective graphs share a point in common?

2x + 7y = -1, 5x + y = 14, 3x + 2y = k

(A) -14 (B) -8 (C) 1 (D) 7 (E) 10

**Answer.** The first two equations meet at a point: $y = 14 - 5x$, so $-1 = 2x + 7y = 2x + 98 - 35x = 98 - 33x$. So $33x = 98$ and $x = 3$, so $y = 14 - 5(3) = -1$. Then for the last line to pass through $(3, -1)$, we need $k = 3(3) - 2(-1) = 9 - 2 = 7$.

**Question 20.** In the figure below, lines marked the same $(>, >)$ are parallel. What is the sum (in degrees) of the angle values, $a + b + c + d + e + f$, equal to?

(A) 180 (B) 240 (C) 270 (D) 360 (E) 450

**Answer.** The horizontal parallel lines give $e + (180 - d) = 180$, so $e = d$. The other parallel lines give $a = f$ and $b = c$. The small triangle gives $d + f + b = 180$, and since $d + f + b = e + a + c$, the sum we seek is 360.

**Question 21.** What is the $x$-intercept of the line which passes through (4, 5) and is perpendicular to the line with equation $y = 2x + 3$?

(A) -5 (B) -1 (C) 4 (D) 9 (E) 14

**Answer.** The slope of the perpendicular line is $-1/2$, so $y = (-1/2)x + b$, and passes through (4, 5), so $5 = (-1/2)(4) + b = -2 + b$ and so $b = 7.$ This line meets the $x$-axis where $0 = (-1/2)x + 7$, so $x = 14$.

**Question 22.** What are all of the values of $x$ for which $\log_5 x = \log_3 x$?

(A) -5 and 5 (B) 1/5 and 5 (C) 1 and 5 (D) only 5 (E) 0, 1 and 5

**Answer.** If $a = \log_5 x = \log_3 x$, then $a^2 = (\log_5 x)(\log_3 x) = \log_5 3\log_3 x = \log_5 x = a$, so $a = 1$ or $-1$. Solving $\log_5 x = a$ then yields $x = 5$ or $x = 1/5$.

**Question 23.** If $A, B$ and $C$ are constants such that for all values of $x$

$3x^2 + 4x + 2 = (Ax + B)(x + 2) + C(x^2 - 1),$

then what value does $A$ have to be?

(A) 0 (B) -1 (C) 1 (D) $\frac{1}{2}$ (E) 4

**Answer.** Multiplying out, $(Ax + B)(x + 2) + C(x^2 - 1) = A(x^2 + 2x) + Bx + 2B + Cx^2 - C$, so we need $A(x^2 + 2x) + Bx + 2B = 3x^2 + 4x + 2$, and since $A + 2B = 3$, then $B = 4 - 2A$, so $A + 2(4 - 2A) = 3$, which gives $-3A = -3$ and so $A = 1$.

**Question 24.** A test consists of 12 questions. 8 points are awarded for a correct answer, and 3 points are subtracted for an incorrect answer. If all questions are answered, which of the following can be a possible total score?

(A) 74 (B) 75 (C) 76 (D) 77 (E) 78

**Answer.** If $x$ questions are correctly answered, then the score will be $8x - 3(12 - x) = 11x - 36$. So the score, added to 36, must be a multiple of 11. 74 + 36 = 110 fits the bill, none of the other choices do.

**Question 25.** The entries of a $7 \times 7$ magic square array are the integers from 1 to 49. The sum of each row, column, and both major diagonals are all the same. What is the value of this `magic’ sum?

(A) 70 (B) 120 (C) 144 (D) 175 (E) 210

**Answer.** Since the rows have the same sum, the sum of all 49 numbers is 7 times the magic sum. But 1 + 2 + $\cdots$ + 48 + 49 = (1 + 49) + (2 + 48) + $\cdots$ + (24 + 26) + 25 = 24 * 50 + 25 = 1225. Dividing by 7 gives the magic sum of 175.