Analysis Qualifying Exam Syllabus

The analysis Ph.D. Qualifying Examination covers the fundamentals of Real Analysis and its standard applications. The main topics are listed below, together with a few references. This list is intended only as a guide.

**Metric space topology:** countable and uncountable sets, including Cantor’s diagonal arguments; topology of Euclidian space and metric spaces, norms and inner products; parallelogram law; compactness in terms of subsequences and open covers; the Heine-Borel theorem; connected sets;

**Sequences and series:** convergence of sequences in metric spaces; Cauchy sequences and completeness; lim sups and lim infs; comparison, ratio, root, integral tests for convergence of series; absolute and conditional convergence; rearrangements of series; alternating series test; summation by parts;

**Continuity of functions:** continuity and uniform continuity for functions of one or several variables; images of connected sets; intermediate value theorem; images of compact sets; extreme value theorem; monotonic functions;

**Calculus of functions of one variable:** definition of derivative; mean value theorem; intermediate value property for derivatives; Taylor’s theorem; Riemann integrals; fundamental theorem of calculus; improper integrals and Cauchy principal values; functions of bounded variation; Riemann-Stieljes integrals;

**Sequences and series of functions:** uniform convergence; conditions for term-by-term continuity, integration, and differentiation; Weierstrass’s M-test; power series and the radius of convergence; Weierstrass’s approximation theorem; equicontinuity and the Arzela-Ascoli theorem.

**References.**