Math 932-933 Qualifying Exam June, 2001 **Name** ____________________.

1. Let $x(t; a, b)$ denote the solution of the IVP

$$x' = \arctan x, \quad x(a) = b.$$

Without solving the above differential equation, find

$$\frac{\partial x}{\partial b}(t; 0, 0) \quad \text{and} \quad \frac{\partial x}{\partial a}(t; 0, 0).$$

Use one of your answers to approximate $x(t; 0, h)$ for h close to zero.

2. State and prove the Picard-Lindeloff Theorem.

3. State and prove the Contraction Mapping Theorem.

4. Approximate the solution of the IVP

$$x' = \frac{x}{1 + x^2}, \quad x(0) = 1$$

by finding the second Picard iterate $x_2(t)$ and find how good an approximation you get.

5. Show that the matrix norm induced by the traffic norm on $\mathbb{R}^n$ is given by

$$\|A\| = \max_{1 \le j \le n} \sum_{i=0}^{n} |a_{ij}|.$$

Recall that the traffic norm on $\mathbb{R}^n$ is defined by

$$\|x\| = \sum_{k=1}^{n} |x_k|.$$

6. Find the Green's function for the BVP

$$x'' = 0, \quad x(0) = 0, \quad x(1) - 2x'(1) = 0.$$

Use your answer to solve the BVP

$$x'' = t, \quad x(0) = 1, \quad x(1) - 2x'(1) = 0.$$