1. Let \( x(t;a,b) \) denote the solution of the IVP

\[
x' = \arctan x, \quad x(a) = b.
\]

Without solving the above differential equation, find

\[
\frac{\partial x}{\partial b}(t;0,0) \quad \text{and} \quad \frac{\partial x}{\partial a}(t;0,0).
\]

Use one of your answers to approximate \( x(t;0,h) \) for \( h \) close to zero.

2. State and prove the Picard-Lindelöf Theorem.


4. Approximate the solution of the IVP

\[
x' = \frac{x}{1+x^2}, \quad x(0) = 1
\]

by finding the second Picard iterate \( x_2(t) \) and find how good an approximation you get.

5. Show that the matrix norm induced by the traffic norm on \( \mathbb{R}^n \) is given by

\[
\|A\| = \max_{1 \leq j \leq n} \sum_{i=0}^{n} |a_{ij}|.
\]

Recall that the traffic norm on \( \mathbb{R}^n \) is defined by

\[
\|x\| = \sum_{k=1}^{n} |x_k|.
\]

6. Find the Green's function for the BVP

\[
x'' = 0, \quad x(0) = 0, \quad x(1) = 2x'(1) = 0.
\]

Use your answer to solve the BVP

\[
x'' = t, \quad x(0) = 1, \quad x(1) = 2x'(1) = 0.
\]