REAL JACOBIAN CONJECTURE AND CUBIC LINEAR MAPPINGS

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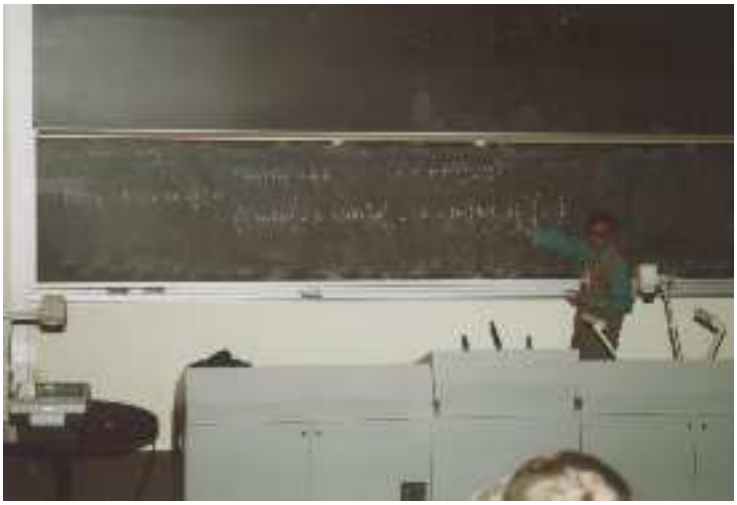

Abstract. Let A be a real square matrix of order n. With each A we associate cubic linear maps, called Drużkowski maps, from $\mathbb{R}$ to $\mathbb{R}$ defined by $F(x) = x+(Ax)$ where the i-th coordinate $F_i(x)$ of $F(x)$ is given by

$$F_i(x) = x_i + (a_{i1}x_1 + \cdots + a_{in}x_n)^3.$$

It is well known that if Drużkowski maps are injective for every A in every dimension n then the well known Jacobian Conjecture holds good. Unfortunately the nonvanishing of the Jacobian alone of the Drużkowski map will not suffice for the injectivity of such maps as shown by an example of Pinchuk.

Here we want to study the injectivity of Drużkowski maps in some special cases by placing some conditions on A. In particular Drużkowski has shown that if the rank of A is n then F is injective. In fact we show that if A is nonsingular (or rank of A is n) then the Jacobian must be a weak P-matrix (or a singular P_0-matrix, all proper principal minors of the Jacobian is nonnegative) and consequently such a Drużkowski map must be injective. Our proof is different from that of Drużkowski.

In the Drużkowski map if the Jacobian of F does not vanish of every $x \in \mathbb{R}^n$, then it puts some restrictions on the matrix A. For example A cannot be an N-matrix

or almost P or $A < 0$ etc. In view of Pinchuk's example it is not clear how useful this information will be. In fact it will be nice to know the rank of A in Pinchuk's example. (Recently professor Engelbert Hubbers has made some progress in this connection and he has given a bound for the dimension n in Pinchuk's example.)

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