A MOUNTAIN PASS TO THE JACOBIAN CONJECTURE

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Abstract. This paper presents a new injectivity theorem and a new open question. The main result of the paper is proved by means of the Mountain Pass Lemma and states that if all the eigenvalues of $F'(\mathbf{x})F'(\mathbf{x})^T$ are bounded away from zero for all $\mathbf{x} \in \mathbb{R}^n$, where $F : \mathbb{R}^n \to \mathbb{R}^n$ is a class C^1 map, then F is injective. This was discovered in a joint attempt by the authors to prove a stronger result conjectured by the first author: Namely, that a sufficient condition for injectivity of class C^1 maps F of $\mathbb{R}^n$ into itself is that all the eigenvalues of $F'(\mathbf{x})$ are bounded away from zero on $\mathbb{R}^n$. If true, it would imply (via *Reduction-of-degree*) *injectivity of polynomial maps* $F : \mathbb{R}^n \to \mathbb{R}^n$ *satisfying the hypothesis*, $\det F'(\mathbf{x}) \equiv 1$, of the celebrated Jacobian Conjecture of Ott-Heinrich Keller. The paper ends with several examples to illustrate a variety of cases and known counterexamples to some natural questions.

References

1. M. Chamberland and G.H. Meisters. *A Mountain Pass to the Jacobian Conjecture* to appear in the Canadian Mathematical Bulletin.