

(1) (a) (This part is #32 on p. 535; 8 points.)  $\sin^{-1}(\sin(3\pi/4)) = \sin^{-1}(\sqrt{2}/2) = \pi/4$ .

(b) (This part is like #7 on p. 542; 12 points.) Find the exact value of  $g'(1)$ , given  $g(x) = \tan^{-1}(f(x))$ ,  $f(1) = 2$  and  $f'(1) = 3$ : using the chain rule  $g'(x) = \frac{1}{1+(f(x))^2} f'(x)$ , so  $g'(1) = \frac{1}{1+(f(1))^2} f'(1) = \frac{1}{1+2^2} 3 = 3/5$ .

(2) (This problem is like #7, p. 566.) Use  $u = \ln(x)$ ,  $dv = x^n dx$ , so  $v = x^{n+1}/(n+1)$  and  $du = dx/x$ ;  $\int x^n \ln(x) dx = (\ln(x))x^{n+1}/(n+1) - \int x^n/(n+1) dx = (\ln(x))x^{n+1}/(n+1) - x^{n+1}/(n+1)^2 + C$ .

(3) (This problem is like #19, p. 585.) First,  $\frac{x+1}{x^3+x} = \frac{Ax+B}{x^2+1} + \frac{C}{x}$ , or  $x+1 = (Ax^2+Bx)+C(x^2+1)$ , so  $C = 1$ ,  $A = -1$ , and  $B = 1$ . Now  $\int \frac{x+1}{x^3+x} dx = \int \frac{-x}{x^2+1} dx + \int \frac{1}{x^2+1} dx + \int \frac{1}{x} dx = -(1/2)\ln(x^2+1) + \arctan(x) + \ln|x| + C$ .

(4) (This problem is like #23, p. 560.) First,  $\int \frac{1}{x^2+6x+13} dx = \int \frac{1}{(x+3)^2+4} dx$ . Now let  $x+3 = 2\tan(t)$ , giving  $\int \frac{1}{(x+3)^2+4} dx = \int \frac{2\sec^2(t)}{4\tan^2(t)+4} dt = \int \frac{2\sec^2(t)}{4\sec^2(t)} dt = (1/2)t + C = (1/2)\arctan((x+3)/2) + C$ .

(5) (This is #52, p. 618.) Note that  $\frac{x^2-2}{x^4+3} \leq \frac{x^2}{x^4+3} \leq \frac{x^2}{x^4} = \frac{1}{x^2}$ , and we know  $\int_1^\infty \frac{1}{x^2} dx$  converges, hence so does  $\int_1^\infty \frac{x^2-2}{x^4+3} dx$ .

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